

**INDIAN SCHOOL MUSCAT**  
**FINAL ASSESSMENT**  
**MATHEMATICS (041)**

**CLASS 12**

1	$\int_{-1}^0 -1 \, dx + \int_0^1 0 \, dx + \int_1^2 1 \, dx = 0$								
2	$\frac{dy}{dx} = e^{(x+y)}$ $\Rightarrow \frac{dy}{dx} = e^x \times e^y$ $\Rightarrow \frac{dy}{e^y} = e^x \, dx$ $\Rightarrow e^{-y} dy = e^x \, dx$ <p><i>Integrate on both side, we have,</i></p> $\int e^{-y} dy = \int e^x \, dx$ $\Rightarrow -e^{-y} = e^x + c$ $\Rightarrow e^x + e^{-y} = k \quad (\text{let } k = -c)$ <p><i>Hence, </i><math>e^x + e^{-y} = k</math></p>								
3	$\hat{a} = \frac{1}{29}(3\hat{i} + 4\hat{j} - 2\hat{k})$ $13\hat{a} = \frac{13}{\sqrt{29}}(3\hat{i} + 4\hat{j} - 2\hat{k})$								
4	$6x - 3y + 2z = 4$ <p>Distance of given point from the plane <math>6x - 3y + 2z - 4 = 0</math></p> $= \left  \frac{6(2) - 3(5) + 2(-3) - 4}{\sqrt{6^2 + 3^2 + 2^2}} \right  = \frac{13}{7}$								
5	i) $\frac{3}{4}$ ii) $\frac{1}{4}$								
6	<p><math>\therefore X</math> can take values 0, 1 or 2.</p> $P(X = 0) = \frac{^{39}C_2}{^{52}C_2} = \frac{39 \times 38}{52 \times 51} = \frac{19}{34}$ $P(X = 1) = \frac{^{13}C_1 \times ^{39}C_1}{^{52}C_2} = \frac{13 \times 39 \times 2}{52 \times 51} = \frac{13}{34}$ $P(X = 2) = \frac{^{13}C_2}{^{52}C_2} = \frac{13 \times 12}{52 \times 51} = \frac{2}{34} = \frac{1}{17}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>0</td><td>1</td><td>2</td></tr> <tr> <td>P(x)</td><td>19/34</td><td>13/34</td><td>2/34</td></tr> </table>	x	0	1	2	P(x)	19/34	13/34	2/34
x	0	1	2						
P(x)	19/34	13/34	2/34						
7	$\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$ $\int \frac{x \, dx}{(x-1)(x-2)} dt = \int \frac{-1}{(x-1)} \, dx + \int \frac{2}{(x-2)} \, dx$ $= -\log x-1  + 2\log x-2  + C$ $= -\log x-1  + \log x-2 ^2 + C$ $= \log \left  \frac{(x-2)^2}{x-1} \right  + C$								

<p><b>8</b></p> $(x^2 - y^2) dx + 2xy dy = 0$ $\frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy} \quad \dots(i)$ <p>It is homogeneous differential equation.</p> <p>Putting <math>y = ux \Rightarrow u + \frac{xdu}{dx} = \frac{dy}{dx}</math></p> <p>From (i) <math>u + x \frac{du}{dx} = -x^2 \frac{(1-u^2)}{2x^2u} = -\left(\frac{1-u^2}{2u}\right)</math></p> $\Rightarrow \frac{xdu}{dx} = -\left[\frac{1-u^2}{2u} + u\right]$ $\Rightarrow \frac{xdu}{dx} = -\left[\frac{1+u^2}{2u}\right]$ $\Rightarrow \frac{2u}{1+u^2} du = -\frac{dx}{x}$	<p>Integrating both sides, we get</p> $\Rightarrow \int \frac{2udu}{1+u^2} = -\int \frac{dx}{x}$ $\Rightarrow \log 1+u^2  = -\log x  + \log C$ $\Rightarrow \log\left \frac{x^2+y^2}{x^2}\right   x  = \log C$ $\Rightarrow \frac{x^2+y^2}{x} = C$ $\Rightarrow x^2+y^2 = Cx$ <p>Given that <math>y=1</math> when <math>x=1</math></p> $\Rightarrow 1+1=C \Rightarrow C=2.$ <p><math>\therefore</math> Solution is <math>x^2+y^2 = 2x.</math></p>
<p><b>9</b></p> <p>We can write,</p> $ \hat{A} - \hat{B} ^2 =  \hat{A} ^2 +  \hat{B} ^2 - 2 \hat{A}  \hat{B}  \cos \theta$ $\Rightarrow  \hat{A} - \hat{B} ^2 = 1 + 1 - 2 \cos \theta$ $\Rightarrow  \hat{A} - \hat{B} ^2 = 2(1 - \cos \theta)$ $\Rightarrow  \hat{A} - \hat{B} ^2 = 4 \sin^2 \frac{\theta}{2} \quad \left(\because 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}\right)$ $\Rightarrow  \hat{A} - \hat{B}  = 2 \sin \frac{\theta}{2}$ <p>OR</p> <p><math>\mathbf{P} = \mathbf{a} \times \mathbf{b} = 32\hat{i} - \hat{j} - 14\hat{k}</math></p> <p><math>\mathbf{d} = \lambda \mathbf{p} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}</math></p> <p><math>\mathbf{c} \cdot \mathbf{d} = 15</math></p> $(2\hat{i} - \hat{j} + 4\hat{k}) \cdot (32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}) = 15$ $\lambda = 5/3$ $\mathbf{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}$	
<p><b>10</b></p> <p>Shortest distance between the lines with vector equations</p> $\text{is } \left  \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{ \vec{b}_1 \times \vec{b}_2 } \right $ $\vec{a}_1 = 1\hat{i} + 2\hat{j} + 3\hat{k} \quad \vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$ $\vec{b}_1 = 1\hat{i} - 3\hat{j} + 2\hat{k} \quad \vec{b}_2 = 2\hat{i} + 3\hat{j} + 1\hat{k}$ $(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 3\hat{j} + 9\hat{k}$ $\left  (\vec{b}_1 \times \vec{b}_2) \right  = \sqrt{(-9)^2 + 3^2 + 9^2} = \sqrt{171} = 3\sqrt{19}$ <p>So, shortest distance = <math>\frac{3}{\sqrt{19}}</math></p>	

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$$\text{Let } I = \int_1^4 [|x - 1| + |x - 2| + |x - 3|] dx$$

$$\Rightarrow I = \int_1^4 |x - 1| dx + \int_1^4 |x - 2| dx + \int_1^4 |x - 3| dx$$

$$I = I_1 + I_2 + I_3 \dots \quad (1)$$

$$\text{where, } I_1 = \int_1^4 |x - 1| dx, I_2 = \int_1^4 |x - 2| dx, \text{ and } I_3 = \int_1^4 |x - 3| dx$$

$$I_1 = \int_1^4 |x - 1| dx$$

$$(x - 1) \geq 0 \text{ for } 1 \leq x \leq 4$$

$$\therefore I_1 = \int_1^4 (x - 1) dx$$

$$\Rightarrow I_1 = \left[ \frac{x^2}{2} - x \right]_1^4$$

$$\Rightarrow I_1 = \left[ 8 - 4 - \frac{1}{2} + 1 \right] = \frac{9}{2} \dots \quad (2)$$

$$\Rightarrow I_3 = [6 - 4] + \left[ \frac{1}{2} \right] = \frac{5}{2} \dots \quad (4)$$

From equations (1), (2), (3), and (4), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

$$I_2 = \int_1^4 |x - 2| dx$$

$x - 2 \leq 0$  for  $2 \leq x \leq 4$  and  $x - 2 \geq 0$  for  $1 \leq x \leq 2$

$$\therefore I_2 = \int_1^2 (2 - x) dx + \int_2^4 (x - 2) dx$$

$$\Rightarrow I_2 = \left[ 2x - \frac{x^2}{2} \right]_1^2 + \left[ \frac{x^2}{2} - 2x \right]_2^4$$

$$\Rightarrow I_2 = [4 - 2 - 2 + \frac{1}{2}] + [8 - 8 - 2 + 4]$$

$$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2} \dots \quad (3)$$

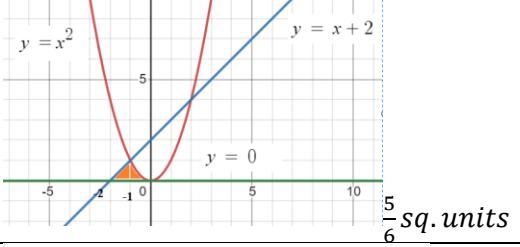
$$I_3 = \int_1^4 |x - 3| dx$$

$x - 3 \geq 0$  for  $3 \leq x \leq 4$  and  $x - 3 \leq 0$  for  $1 \leq x \leq 3$

$$\therefore I_3 = \int_1^3 (3 - x) dx + \int_3^4 (x - 3) dx$$

$$\Rightarrow I_3 = \left[ 3x - \frac{x^2}{2} \right]_1^3 + \left[ \frac{x^2}{2} - 3x \right]_3^4$$

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5 sq. units

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The equation of the plane passing through three given points can be given by

$$\begin{vmatrix} x-2 & y-2 & z-1 \\ x-3 & y-0 & z-1 \\ x-4 & y+1 & z-0 \end{vmatrix} = 0$$

Performing elementary row operations R2 → R1-R2 and R3 → R1-R3, we get

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 3-2 & 0-2 & 0 \\ 4-2 & -1-2 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 1 & -2 & 0 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow x=(-\lambda+3), y=(\lambda-4), z=(6\lambda-5)$$

At the point of intersection, these points satisfy the equation of the plane  $2x+y+z-7=0$ .

Putting the values of x, y and z in the equation of the plane, we get the value of  $\lambda$ .

$$2(-\lambda+3)+(\lambda-4)+(6\lambda-5)-7=0$$

$$\Rightarrow -2\lambda+6+\lambda-4+6\lambda-5-7=0$$

$$\Rightarrow 5\lambda=10$$

$$\Rightarrow \lambda=2$$

Thus, the point of intersection is P(1, -2, 7).

$$\Rightarrow (2x-4)+(y-2)+(z-1)=0$$

$$\Rightarrow 2x+y+z-7=0$$

Therefore, the equation of the plane is  $2x+y+z-7=0$

Now, the equation of the line passing through two given points is

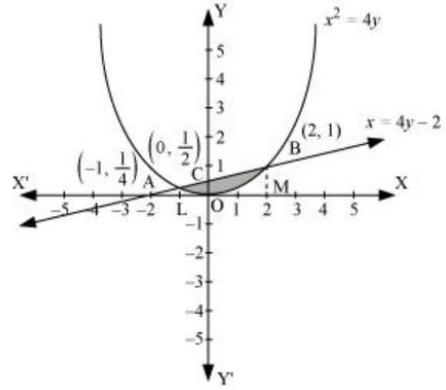
$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} = \lambda$$

$$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$$

$$\Rightarrow x=(-\lambda+3), y=(\lambda-4), z=(6\lambda-5)$$

At the point of intersection, these points satisfy the equation of the plane  $2x+y+z-7=0$ .

<p>14.</p> <p><math>P(A_1) = \frac{4}{10}</math>, <math>P(A_2) = \frac{4}{10}</math> and <math>P(A_3) = \frac{2}{10}</math></p> <p>where <math>A_1</math>, <math>A_2</math> and <math>A_3</math> denote the three types of flower seeds.</p> <p>Let <math>E</math> be the event that a seed germinates and <math>E'</math> be the event that a seed does not germinate.</p> <p><math>\therefore P(E/A_1) = \frac{45}{100}</math>, <math>P(E/A_2) = \frac{60}{100}</math> and <math>P(E/A_3) = \frac{35}{100}</math></p> <p>and <math>P(E'/A_1) = \frac{55}{100}</math>, <math>P(E'/A_2) = \frac{40}{100}</math> and <math>P(E'/A_3) = \frac{65}{100}</math></p> <p>(i) <math>\therefore P(E) = P(A_1) \cdot P(E/A_1) + P(A_2) \cdot P(E/A_2) + P(A_3) \cdot P(E/A_3)</math></p> $= \frac{4}{10} \cdot \frac{45}{100} + \frac{4}{10} \cdot \frac{60}{100} + \frac{2}{10} \cdot \frac{35}{100}$ $= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = 0.49$	
<b>SET B</b>	
<p>1. 0</p> <p>7. <math>\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}</math></p> <p><math>A = -1</math>, <math>B = 2</math></p> <p><math>\int \frac{x}{(x+1)(x+2)} dx</math></p> $= -\log x+1  + \log x+2 ^2 + C$ $= \log \frac{(x+2)^2}{ x+1 } + C$	
<p>13. <math>\frac{x-2}{3} = \frac{y-3}{-1} = \frac{z-7}{-1} = r</math> (say)</p> <p>Foot of perpendicular is <math>(3r+2, -r+3, -r+7)</math>.</p> <p>Substituting this in the coordinates of M, we get</p> <p><math>M = (3r+2, -r+3, -r+7) = (3(1)+2, -1+3, -1+7) = (5, 2, 6)</math></p> <p>Image is <math>(8, 1, 5)</math></p>	
<b>SET C</b>	
<p>5</p> <p>i) <math>P = 1/10</math></p> <p>ii) <math>P = 1/5</math></p>	
<p>10</p> $(\vec{a}_2 - \vec{a}_1) = 0\hat{i} + 1\hat{j} - 4\hat{k}$ $(\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$ $= 2\hat{i} - 4\hat{j} - 3\hat{k}$ $ \vec{b}_1 \times \vec{b}_2  = \sqrt{4 + 16 + 9} = \sqrt{29}$ $(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (2\hat{i} - 4\hat{j} - 3\hat{k}) \cdot (0\hat{i} + 1\hat{j} - 4\hat{k})$ $= 8$ <p>So, shortest distance = <math>\left  \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{ \vec{b}_1 \times \vec{b}_2 } \right  = \frac{8}{\sqrt{29}}</math></p>	



$$\int_{-1}^2 \left( \frac{x+2}{4} - \frac{x^2}{4} \right) dx$$

$$\frac{1}{4} \left( \frac{x^2}{2} + 2x - \frac{x^3}{3} \right)$$

= 9/8 sq.units

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- 1) 11/15  
2) 9/11